

Unit Review

Topics:

- Periodic Functions – definitions (periodic function, amplitude, cycle, period)
- Sketching $y = \sin x$ and $y = \cos x$
- Transformations of periodic functions in the form $y = af(k(x - d)) + c$ where $f(x) = \sin x$ or $f(x) = \cos x$
- Describing key properties of periodic functions: domain, range, maximum value, minimum value, amplitude, intercepts, period, intervals of increase/decrease
- Determining the equation of trigonometric functions
- Applications of trigonometric functions – modeling periodic phenomena

Make sure you have a complete set of notes and have done all the homework!

Intro to Periodic Functions & Definitions (textbook 5.3):

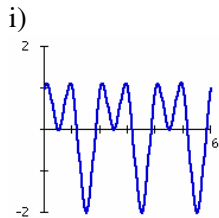
Textbook questions: Pg. 414 #15, 16

Supplementary problems:

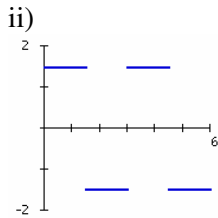
1) Classify each of the following graphs as periodic or not periodic.

If the graph is periodic:

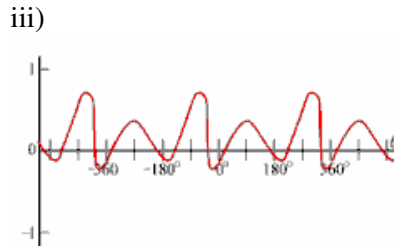
- highlight a cycle
- determine the period, amplitude, axis of the curve, and the maximum and minimum value



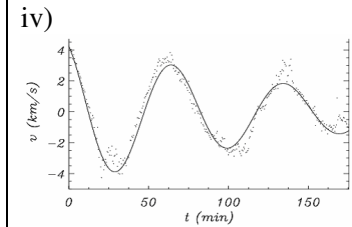
Evaluate $f(7)$



Evaluate $f(33)$



Evaluate $f(-720^\circ)$



Evaluate $f(1000)$

2) Explain why each case could be an example of a periodic phenomenon

- the position of a pendulum in a grandfather clock
- the vibration of a guitar string
- the number of litres of ice cream consumed in Ontario each year

Graphing Sinusoidal Functions (5.5 & 5.6):

Textbook questions Pg. 414 – 415 #19, 20 (ignore radians), 22, 25 (a and b), 26

Supplementary problems:

3) Describe key properties of the graphs of $y = \sin x$ and $y = \cos x$: domain, range, maximum value, minimum value, amplitude, intercepts, period, intervals of increase/decrease.

4) Describe each of the following transformations on $y = \sin x$:

- $y = \sin 3x$
- $y = \frac{1}{2} \sin x + 1$
- $y = 2 \sin(x - 45^\circ)$

5) Describe each of the following transformations on $y = \cos x$:

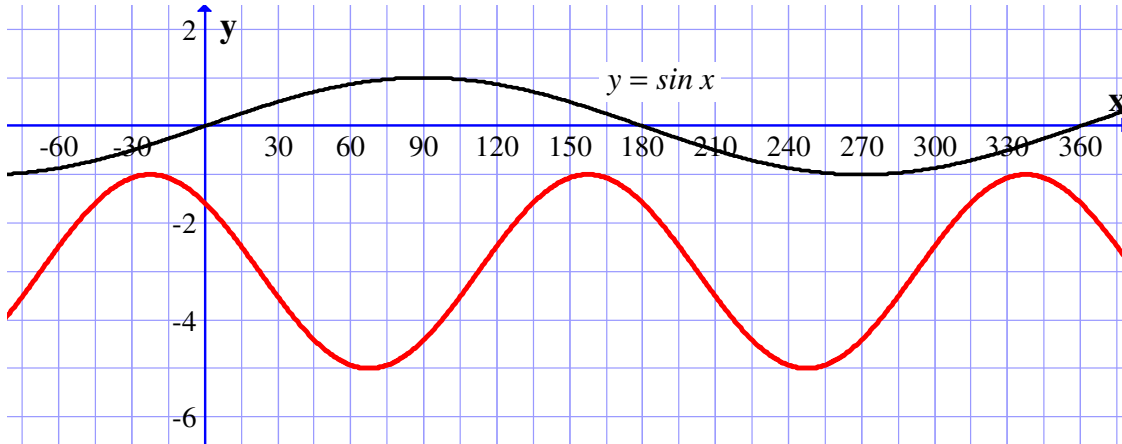
- $y = \cos \frac{3}{2} x$
- $y = \cos 2(x - 45^\circ)$
- $y = -2 \cos(3x + 180^\circ) + 2$

6) Sketch one cycle of each of the graphs from #4 and #5. Label key points.

Determining Equations of Sinusoidal Functions (5.5 & 5.6):

Supplementary Problems:

- 7) Write an equation for the function with the following characteristics:
- transformation of $y = \cos x$, amplitude = 5, period = 180° , axis of curve $y = 1$, phase shift 90° left
 - a sinusoidal function has an amplitude of 4, a period of 120° , and a maximum at $(30^\circ, 3)$
 - the transformation of $y = \sin x$ shown below:

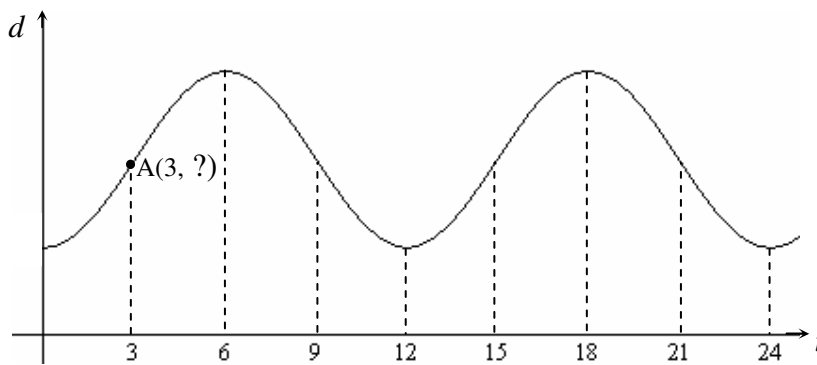


Modelling Periodic Behaviour (5.5 & 5.6):

Textbook questions Pg. 515 – 516 #24, 30, 31

Supplementary Problems:

- 8) Because of the tide, the depth of the water in a harbour is modelled by the equation $d(t) = -3\cos(30t)^\circ + 6$, where d represents the depth of the water in metres and t represents the number of hours after midnight. (i.e. $t = 0$ means midnight, $t = 1$ means 1 A.M., and so on.) The graph of the relation is shown below:

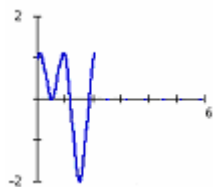


- What is the missing coordinate of point A? What do the coordinates of point A represent?
 - State the maximum depth of the water.
 - Surfing is allowed between 8 A.M. (08:00 hrs) and 7 P.M. (19:00 hrs), but only when the depth of the water is 6 m or more. For how many hours is surfing allowed in one day? Explain.
- 9) A Ferris wheel with a radius of 9.5 m rotates once every 10 s. The bottom of the wheel is 1.2 m above the ground.
- Find the equation of the sine function that gives a rider's height above the ground, in metres, as a function of the time, in seconds, with the rider starting at the bottom of the wheel.
 - Sketch the graph showing two complete cycles.

Supplementary problem Answers:

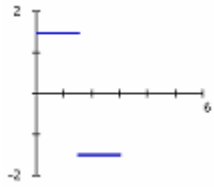
1)

i) a) highlighted portion may vary



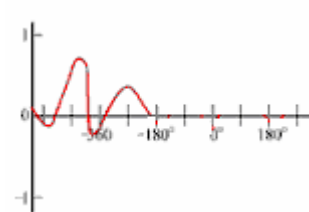
$Period = 2$
 $Amplitude = 1.5$
 $Axis\ of\ curve: y = -0.5$
 $Max.\ value = 1$
 $Min.\ value = -1.5$
 $f(7) = f(7 - 2 \cdot 3)$
 $= f(7 - 6)$
 $= f(1)$
 $= 1$

ii) a) highlighted portion may vary



$Period = 3$
 $Amplitude = 1.5$
 $Axis\ of\ curve: y = 0$
 $Max.\ value = 1.5$
 $Min.\ value = -1.5$
 $f(33) = f(33 - 3 \cdot 11)$
 $= f(33 - 33)$
 $= f(0)$
 $= 1.5$

iii) a) highlighted portion may vary



$Period = 360^\circ$
 $Amplitude = 0.5$
 $Axis\ of\ curve: y = 0.3$
 $Max.\ value = 0.8$
 $Min.\ value = -0.2$
 $f(-720^\circ) = f(-720^\circ + 360^\circ)$
 $= f(-360^\circ)$
 $\doteq -0.15$

iv)

Not periodic

$f(1000)$ value cannot be predicted

2) a) The pendulum swings from a point to the right of vertical, through the vertical position to a point on the left of vertical. It then swings back to its starting point.

b) When the string is plucked it moves away from rest, then back through rest to the other side. This vibration, over a short period of time, could be considered as periodic although it will eventually stop!

c) Ice cream consumption follows a regular pattern as it increases from a low in the cooler months to a high in the warmer months and then decreases as the temperature cools. This cycle repeats each year.

3) Look back at the table created in class for Lesson 4: Connecting Trigonometric Functions to the Properties.

4) a) horizontal compression, factor is 3

b) vertical compression, factor is 2; vertical translation up 1 unit

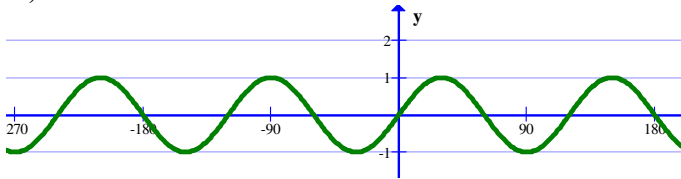
c) vertical stretch, factor is 2; horizontal translation right by 45°

5) a) horizontal compression, factor is 1.5

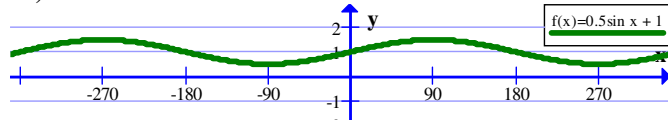
b) horizontal compression, factor is 2; Phase Shift = 45° (horizontal translation right)

c) reflection in x-axis; vertical stretch, factor is 2; horizontal compression, factor is 3; Phase Shift = -60° (horizontal translation left); vertical translation up 2 units

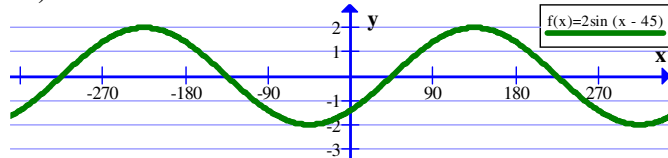
6) 4a)



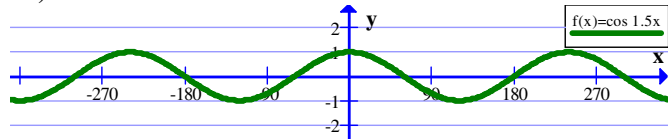
4b)



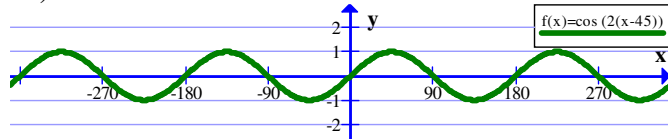
4c)



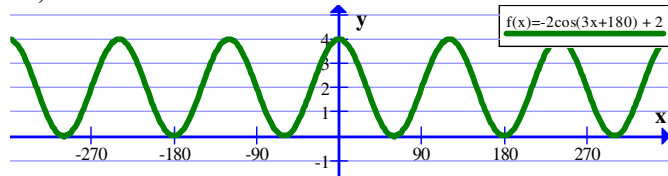
5a)



5b)



5c)



7) a) $y = 5 \cos[2(x + 90)]^\circ + 1$

b) $y = 4 \sin(3x) - 1$ or $y = 4 \cos[3(x - 30^\circ)] - 1$

c) $y = -2 \sin(2x - 45^\circ) - 3$

8) a) $A(3, 6)$ represents that at 3 A.M. the depth of water is 6 m

b) 9 m

c) 8 A.M. to 9 A.M. then 3 P.M. to 7 P.M. therefore 5 hours per day.

9) a) $y = 9.5 \sin[36(x - 2.5)]^\circ + 10.7$ or $y = 9.5 \sin[36(x + 7.5)]^\circ + 10.7$

b) Sketch the graph showing two complete cycles.

